Some Operators for Refinement of Normative Systems

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Abstract. Refinement is an operation of theory change where new or old beliefs (norms) are restricted by some specific condition, instead of being fully accepted or rejected. We present constructions, based on AGM partial meet contraction and revision, for operators of external, internal and global refinement and show that they satisfy reasonable AGM-like postulates. Then we apply these operators to a hard case and show that global refinement has advantages to cope with the problem of gaps in normative systems.

1 Introduction

The identification of the relevant conditions to the enforcement of legal rules is a problem, which, according to H.L.A. Hart, lies in the very core of legal reasoning:

"in the case of legal rules, the criteria of relevance and closeness of resemblance depend on many complex factors running through the legal system and on the aims or purpose which may be attributed to the rule."..."to characterize these would be to characterize whatever is specific or peculiar in legal reasoning."(9, p.127).

An attempt to solve this problem would be to define as *relevant* those conditions which are explicitly mentioned by a norm in the normative system as a condition of application of a normative solution (obligatory, forbidden, permitted). This would be, according to Alchourrón and Bulygin (1, p.154), a description of the *thesis of relevance* of the normative system. But in legal cases there may be circumstances in which the normative solution provided by the legal system violates the (moral or political) purposes of the regulation. Or at least there may be circumstances in which the opposite normative solution to that indicated by the system seems to further these purposes better. Clearly, these circumstances are, in some sense, *relevant*, even though they were unforeseen by the lawgiver. According to Alchourrón and Bulygin, they would belong to a description of the *hypothesis of relevance* of the normative system, which always express a value judgement about its axiological adequacy (1, p.156-157). If a condition which *ought to* be relevant according to the hypothesis of relevance is not present in the descriptive thesis of relevance of the normative system, they speak of an *axiological gap*. An axiological gap should then be distinguished from a *normative gap* which is present

Juliano S. A. Maranhão, 'Some Operators for Refinement of Normative Systems' in Bart Verheij, Arno R. Lodder, Ronald P. Loui and Antoinette J. Muntjewerff (eds.), *Legal Knowledge and Information Systems. Jurix 2001: The Fourteenth Annual Conference*. Amsterdam: IOS Press, 2001, pp. 103-115.

when the lawgiver does not provide any normative solution to a relevant case (according to the thesis of relevance).

For example, in a regulation constituted by the single norm “it is forbidden to smoke in this room”, the thesis of relevance describes the condition of “being inside the room” as relevant. But one may argue that according to the purpose of the rule, which is, say, “non disturbance”, smoking in the room should be permitted, if the smoker is next to an open window. This last condition is in the hypothesis of relevance and since the proposed solution conflicts with the non-smoking rule, it constitutes an axiological gap.

In a recent paper (13), Rodriguez claims that the hypothesis of relevance and eventual axiological gaps should not be immediately identified with prescriptive (axiological) discourse, as they may be understood as conveying information about a second normative system which is actually a reconstruction, by the legal interpreter, of the so-called *mens legislatoris*. This descriptive account gives expression to a well-known argument employed in claims for axiological gaps, according to which, the legislator would have provided a different solution had he known the unpredictable circumstance. So the new solution, which is supposed to fill the gap, is provided as a description, in the name of the (rational and fair) legislator, of a qualified normative system.

It is then clear that, in such reconstructions, the interpreter, in a concealed way, *changes* the original normative system, providing new normative solutions to hold in “unforeseen” cases and, accordingly, introducing new conditions to support old normative solutions. Let us call *refinement* this dynamic process of change by qualification of norms (or beliefs). So an important step in pursue of Hart’s *desideratum* to characterize reasoning about relevant normative conditions consists in investigating the rationality of refinement. In the case of an axiological gap, the interpreter should solve the conflict between the old and the proposed solution remaining as faithful as possible to the lawgiver’s will. Thus, it is desirable that the resulting refined system be *consistent* and that the modification of the original system be *minimal*. How should we then define refinement operations in order to satisfy these demands?

Minimal and consistent belief change is the object of concern of Belief Revision. But standard models in this field, such as the AGM model (3), and even recent non-prioritized approaches (8), are not suited to the present purposes, since in these models conflicting information (norm), be it new or old, is *fully* rejected and not restricted by some condition as in refinement. But refinement operators can be developed out of the AGM model. In (11), an operator of belief change called *external refinement* was proposed to model processes where a condition is introduced to restrict the new (input) information, in case it conflicts with one of the original beliefs. In the present work, some new operators of refinement are introduced. In the operation called *internal refinement*, the agent, when confronted with conflicting new information, introduces a condition to one of his original beliefs. In the operation called *global refinement* both new and old beliefs are restricted by corresponding conditions. These operators are then applied to a (hard) antitrust case and it is shown that global refinement has advantages to cope with normative gaps in refined normative systems.

2 The AGM Partial Meet Contraction and Revision Operators.

In the AGM model (3), beliefs of an agent are represented by a logically closed set K of formulas in propositional language $\mathcal{L}$. That is, given an inference relation $\vdash$ betweeen sets of formulas and formulas in $\mathcal{L}$, a Tarskian consequence operator on sets of formulas Cn is defined as $Cn(\Gamma) = \{\alpha \in \mathcal{L} : \Gamma \vdash \alpha\}$. Cn is assumed to satisfy self-deductibility, monotonicity and idempotence. A set $\Gamma \subseteq \mathcal{L}$ is logically closed iff $\Gamma = Cn(\Gamma)$. We also call logically closed sets theories. The expansion of a theory K by α is represented by $K + \alpha = Cn(K \cup \{\alpha\})$.

The AGM model is governed by the following postulates for theory contraction, which are suppose to capture the underlying intuitions for rationally giving up beliefs (the contraction of K by α is denoted by $K \div \alpha$):

K÷1) *ct-closure*: $K \div \alpha$ is a theory
K÷2) *ct-success*: if $\nvdash \alpha$ then $\alpha \notin K \div \alpha$
K÷3) *ct-inclusion*: $K \div \alpha \subseteq K$
K÷4) *ct-vacuity*: if $\alpha \notin K$ then $K \div \alpha = K$
K÷5) *ct-recovery*: $K \subseteq (K \div \alpha) + \alpha$
K÷6) *ct-extensionality*: if $\vdash \alpha \leftrightarrow \beta$ then $K \div \alpha = K \div \beta$

To accept information which contradicts previous beliefs demands revision (the revision of K by α is denoted by $K * \alpha$), which should satisfy the following postulates:

K∗1) *r-closure*: $K * \alpha$ is a theory
K∗2) *r-success*: $\alpha \in K * \alpha$
K∗3) *r-inclusion*: $K * \alpha \subseteq K + \alpha$
K∗4) *r-vacuity*: if $K \nvdash \alpha$ then $K + \alpha \subseteq K * \alpha$
K∗5) *r-consistency*: if $\nvdash \neg\alpha$ then $K * \alpha \neq \mathcal{L}$
K∗6) *r-extensionality*: if $\vdash \alpha \leftrightarrow \beta$ then $K * \alpha = K * \beta$

While Gärdenfors (7) suggested the above postulates for belief change, Alchourrón and Makinson (2) provided explicit definitions of contraction functions modelling such processes.

Definition 1. *Le Γ be a set of formulas and α a formula. The remainder set $\Gamma \perp \alpha$ of Γ is the set of subsets Δ of Γ such that:*
(i) $\Delta \nvdash \alpha$
(ii) there is no Δ' such that $\Delta \subset \Delta' \subseteq \Gamma$ and $\Delta' \nvdash \alpha$

As a tool for choosing elements from the remainder set, AGM introduce a selection function f.

Definition 2. *Let Δ be a set of formulas. A selection function for Δ is a function f such that for all sentences α:*
(a) If $\Delta \perp \alpha \neq \emptyset$, then $f(\Delta \perp \alpha)$ is a non-empty subset of $\Delta \perp \alpha$.
(b) If $\Delta \perp \alpha = \emptyset$, then $f(\Delta \perp \alpha) = \{\Delta\}$.

The *partial meet contraction* of a theory K by α is then given by the intersection of the elements chosen by the selection function, i.e. $K \div_f \alpha = \bigcap f(\Delta \perp \alpha)$.

A revision of a belief set K by α, is then the contraction of K by $\neg\alpha$ composed with its expansion by α: $K * \alpha = (K \div \neg\alpha) + \alpha$. This is the so-called *Levi identity*.

One of the main results of the AGM classical paper (3) was the proof that the Gärdenfors postulates are axiomatic characterizations of Alchourrón and Makinson's constructions, that is, let K be a theory:

Theorem 1. *An operator $\div$ on K is a* partial meet contraction *iff it satisfies K$\div$1-6.*

Theorem 2. *An operator $*$ on K is a* partial meet revision *iff it satisfies K$*$1-6.*

3 The Conditionalization Function

As the main tool to the construction of external, internal and global refinement we introduce the conditionalization function c. Intuitively, the conditionalization function does the work of finding the proper condition to a considered new (original) information in the process of external (internal) refinement. As we are not interested here in the reasons by which agents choose this or that condition, any formula may be chosen.

Definition 3. *A conditionalization function is a function c on the set and into the set of formulas $\mathcal{L}$, such that $c(\alpha) = \beta \rightarrow \alpha$.*

Proposition 3. *The conditionalization function satisfies* implication*:* $\vdash \alpha \rightarrow c(\alpha)$.

The following restraints on the conditionalization function will prove to be useful for the construction of the different operators of refinement:

Definition 4. *A conditionalization function is non-tautological iff, for any formula α, if $\nvdash \alpha$ then $\nvdash c(\alpha)$.*

Definition 5. *A conditionalization function is* maximal *iff, for any α, $\vdash c(\alpha) \leftrightarrow \alpha$.*

Definition 6. *A conditionalization function is* e-maximal *iff, for any α: if $K \nvdash \neg\alpha$ then $\vdash c(\alpha) \leftrightarrow \alpha$.*

Definition 7. *A conditionalization function is* i-maximal *iff, for any α: if $K \nvdash \alpha$ then $\vdash c(\alpha) \leftrightarrow \alpha$.*

Definition 8. *A conditionalization function is* extensional *iff, for any α: if $\vdash \alpha \leftrightarrow \beta$ then $\vdash c(\alpha) \leftrightarrow c(\beta)$.*

Definition 9. *A conditionalization function is* negation invariant *iff, for any α and any β, $c(\alpha) = \beta \rightarrow \alpha$ iff $c(\neg\alpha) = \beta \rightarrow \neg\alpha$.*

4 External Refinement

External refinement is the theory change which is obtained through the acceptance, under a certain condition, of a new conflicting information. Here a condition is stipulated for the information which is coming into the belief set.

External refinement of a theory K by α, an operation which will be denoted by $K \Subset \alpha$ is simply a partial meet revision of K by a refined formula $\beta \rightarrow \alpha$. Given this picture, some basic conditions become plausible.

K$\Subset$1) *e-closure:* $K \Subset \alpha$ is a theory
K$\Subset$2) *e-inclusion:* $K \Subset \alpha \subseteq K + \alpha$
K$\Subset$3) *e-success*: $\beta \rightarrow \alpha \in K \Subset \alpha$

K$\Subset$4) *e-consistency*: if $\nvdash \neg\alpha$ then $K \Subset \alpha \neq \mathcal{L}$
K$\Subset$5) *e-vacuity*: if $K \nvdash \neg\alpha$, then $K \Subset \alpha = K + \alpha$
K$\Subset$6) *e-extensionality*: if $\vdash \alpha \leftrightarrow \beta$, then $K \Subset \alpha = K \Subset \beta$

In the refinement of K by α, the agent is going to accept into our belief set K a formula which is weaker than α, so it is reasonable to expect that the resulting theory will be smaller than the expansion of K by α. Success, in the AGM sense, that $\alpha \in K \Subset \alpha$, will obviously not hold. Indeed the very idea of external refinement is to avoid r-success, thus being much more cautious with the accepted information. However, the conditioned input will always be accepted, so that we obtain a kind of success where the input information is accepted if the condition also holds. For vacuity, if faced with new information which does not conflict with original beliefs, the agent may fully accept it, having no reason for modification. The operation should not be dependent on the logical form of the input, so extensionality should hold.

Using the same strategy as the one employed by Fermé and Hansson (5), the construction of external refinement is straightforward.

Definition 10. *Let K be a theory, $*$ a partial meet revision operator and c a conditionalization function. The external refinement based on c and $*$ is the operation such that, for any α, $K \Subset \alpha = K * c(\alpha)$.*

Theorem 4. *Let $\Subset$ be an operator of external refinement based on a partial meet revision operator $*$ and on a* e-maximal *and* extensional *conditionalization function c. Then $\Subset$ satisfies K$\Subset$1-6.*

In addition to these, the following interesting property is satisfied by this construction. For a theory K and $c(\alpha) = \beta \rightarrow \alpha$:

relative vacuity: if $\neg\alpha \in K$ and $\beta \notin K$, then $K \Subset \alpha = K + (\beta \rightarrow \alpha)$.

Suppose that the new information contradicts our belief set. So we are going to refine it in order to avoid inconsistency. Suppose now that the stipulated condition under which the new information holds is not the case in our original belief state. Then, the contradiction will not arise if we only add the refined input into the belief set. That is the meaning of relative vacuity.

External refinement is a generalization of partial meet revision, since partial meet revision can be defined as refining by use of a maximal conditionalization function.

Definition 11. *Let $\Subset$ be an operator of external refinement, c a maximal conditionalization function and K a theory. Then the operator $*$ of partial meet revision of K by α is the operation such that for every α, $K * \alpha =_{df} K \Subset \alpha$.*

It is easy to see that the operator $*$ thus defined satisfies the r-postulates $K*$1-6. As the reader may notice, external refinement is non-prioritized, that is, that α or $\neg\alpha$ belongs to $K \Subset \alpha$ is open and depends on the choice on the remainder set.

5 Internal Refinement

Internal refinement parallels a process of contraction where some belief will be weakened and others abandoned. The example below shows how internal refinement is intended to work, and its advantages over the AGM model:

Suppose Ciclano believes that water boils at 100^oC (α). Some day he cooks beans at his home in La Paz and notices that water boiled at a temperature below 100^oC ($\neg\alpha$). As he has noticed that pressure conditions are relevant, that is, not being on sea level ($\neg\beta$) is a *refuting condition* for α, his mistake was to believe that *whatever the case may be* water boils at 100^oC, i.e. both in $\beta \rightarrow \alpha$ and $\neg\beta \rightarrow \alpha$. Ciclano may be right about his belief that "on the sea level water boils at 100^oC" ($\beta \rightarrow \alpha$). Nothing in his water boiling experience showed the contrary, but it did show that in presence of the refuting condition water boils at a temperature below 100^oC. So the only mistake was to believe that "if one is not on the sea level then water boils at 100^oC" ($\neg\beta \rightarrow \alpha$). Hence, it would be a misinterpretation of the experiment to perform an AGM revision and to believe in $\neg\alpha$ whatever the case may be, since it would then follow that "on the sea level water boils at a temperature below 100^oC". As there are no grounds for this last statement, the most rational epistemic attitude seems to be a contraction by $\neg\beta \rightarrow \alpha$. We then expect Ciclano's absolute belief in α to be abandoned and the conditional $\beta \rightarrow \alpha$ to be preserved.

In the above example, it is suggested that employing a contraction by $\neg\beta \rightarrow \alpha$ on a theory K that contains α will preserve $\beta \rightarrow \alpha$. This suggestion is warranted by the following lemma:

Lemma 5. *Let $\div$ be an operator of partial meet contraction and K a theory. If $\alpha \in K$, then $\beta \rightarrow \alpha \in K \div (\neg\beta \rightarrow \alpha)$.*

Using the above lemma, a suitable construction for internal refinement using partial meet contraction and the conditionalization function is straightforward:

Definition 12. *Let K be a theory, $\div$ a partial meet contraction operator and c a conditionalization function. The* internal refinement *based on c and $\div$ is the operation such that, for any α, $K \Supset \alpha = K \div c(\alpha)$.*

Let us examine the properties satisfied by the proposed construction of internal refinement.

Theorem 6. *Let K be a theory, $\Supset$ an operator of internal refinement on K and c a conditionalization function. Then $\Supset$ satisfies:*

K⋑1) i-closure: $K \Supset \alpha$ *is a theory*
K⋑2) i-inclusion: $K \Supset \alpha \subseteq K$
K⋑3) i-recovery: $K \subseteq (K \Supset \alpha) + c(\alpha)$

Further plausible properties are satisfied if we introduce restraints on the conditionalization function.

Theorem 7. *If the conditionalization function c is,*
non-tautological*, then $\Supset$ satisfies K⋑4)* i-success: *if* $\nvdash \alpha$ *then* $c(\alpha) \notin K \Supset \alpha$
i-maximal*, then $\Supset$ satisfies K⋑5)* i-vacuity: *if* $\alpha \notin K$ *then* $K \subseteq K \Supset \alpha$.
extensional*, then $\Supset$ satisfies K⋑6)* i-extensionality:
$if \vdash \alpha \leftrightarrow \gamma$ *then* $K \Supset \alpha = K \Supset \gamma$.

From now on, we are going to refer to internal refinement as the operation based on non-tautological, i-maximal and extensional conditionalization function c. It is then a particular case of partial meet contraction.

Theorem 8. *Every internal refinement operator is an operator of partial meet contraction on theories, but the reciprocal does not hold.*

Corollary 9. *Let* $\Supset$ *be an internal refinement operator and* c *an arbitrary conditionalization function such that* $c(\alpha) = \neg\beta \to \alpha$*. Then* $\Supset$ *satisfies* i-preservation*: if* $\alpha \in K$*, then* $\beta \to \alpha \in K \Supset \alpha$*.*

The adequacy of this construction is given by its satisfaction of the Gärdenfors postulates for contraction in addition to its satisfaction of i-success and i-preservation, which captures the idea of rejecting the old belief when the refuting condition is present and accepting it when it is absent.

6 Global Refinement

Global refinement is a composition of internal and external refinement, where the agent will both refine an original belief and accept that under the refuting condition the contrary holds.

Suppose Beltrano is in a room at a hospital and believes, due to a "*do not smoke*" sign fixed at the front door, that it is forbidden to smoke inside ($\neg\alpha$). A person sitting next to an open window starts smoking and he calls the nurse, who, surprisingly, tells him that smoking there is allowed (α). Instead of believing that she is joking (rejecting α) or the sign is outdated (rejecting $\neg\alpha$), he takes both seriously and reinterprets the regulation considering the fact that the smoker is next to an open window (β) as an exceptional condition. So he comes to believe that smoking is permitted if one is next to an open window ($\beta \to \alpha$), *otherwise* smoking is forbidden ($\neg\beta \to \neg\alpha$).

Given this picture, we suggest the following rules to govern the global refinement of our beliefs with respect to a new information α, where β is the condition for acceptance of α and for refuting $\neg\alpha$.

K$\circledcirc$1) *g-closure*: $K \circledcirc \alpha$ is a theory
K$\circledcirc$2) *g-inclusion*: $K \circledcirc \alpha \subseteq K + \alpha$
K$\circledcirc$3) *g-consistency*: if $\nvdash \neg\alpha$ then $K \circledcirc \alpha \neq \mathcal{L}$
K$\circledcirc$4) *g-success*: $\beta \to \alpha \in K \circledcirc \alpha$
K$\circledcirc$5) *g-vacuity*: if $K \nvdash \neg\alpha$ then $K + \alpha \subseteq K \circledcirc \alpha$
K$\circledcirc$6) *g-extensionality*: if $\vdash \alpha \leftrightarrow \beta$ then $K \circledcirc \alpha = K \circledcirc \beta$
K$\circledcirc$7) *g-preservation*: $\neg\alpha \in K$, then $\neg\beta \to \neg\alpha \in K \circledcirc \alpha$

Such rules capture the intuitions underlying our discussion of refinement: the result of global refinement should be a consistent theory (g-closure and g-consistency). If it is globally refined by a belief, then this belief will hold under a certain condition, while its negation will hold under the oposite condition if originally believed unconditionally (g-success and g-preservation). As for the introduction of a weakened information some beliefs must be abandoned, the refined theory is always a subset of the expansion of the original theory by the full information (g-inclusion). If the new information does not conflict with any original beliefs then there is no problem in accepting it (g-vacuity). Refining by equivalent information should yield the same globally refined theory (g-extensionality).

The construction of the global refinement operator is based on internal refinement and the conditionalization function:

Definition 13. *Let K be a theory, $\Supset$ an operator of internal refinement and c a conditionalization function. Then the operation of global refinement of K by α is such that, for any α, $K \circledcirc \alpha = (K \Supset \neg\alpha) + c(\alpha)$.*

Theorem 10. *Let $\circledcirc$ be an operator of global refinement on a theory K, α any formula, and c an arbitrary conditionalization function such that $c(\alpha) = \beta \rightarrow \alpha$. Then $\circledcirc$ satisfies K$\circledcirc$1-4. If c is extensional and e-maximal, then $\circledcirc$ satisfies K$\circledcirc$5-6. If c is also negation invariant, then $\circledcirc$ satisfies K$\circledcirc$7.*

7 A hard case

Let us examine the application of refinement operators to a hard antitrust case. Section 2 of the *Sherman Act* forbids any act which constitutes an “attempt to monopolize”. A later statute, the *Clayton Act*, forbids, in its Section 7, acquisitions which may “substantially lessen competition” or “tend to create a monopoly”. The purpose (rationale) of this norm is to protect competition within the market, which, by its turn, serves further goals like productive market efficiency and welfare of consumers. The table below sums up the antitrust merger regulation by Sherman and Clayton Acts:

ShermanAct(1)/ClaytonAct(7)

Relevant Cases	Normative Solutions
1. market power	Forbidden to merge
2. not market power	Permitted to merge

Suppose there is a market with only two active players which are merging and that the acquired firm is going bankrupt, thus being forced to leave the market if not acquired. May or may not such acquisition be carried out according to Section 7 of the *Clayton Act*?

In a literal interpretation of the statutes we have, closing the regulation under classical consequence, the following solutions with respect to failing firms.

Literal statutory interpretation

Relevant Cases	Normative Solution
1. failing and market power	Forbidden to merge
2. not failing and market power	Forbidden to merge
3. failing and not market power	Permitted to merge
4. not failing and not market power	Permitted to merge

The table shows that if the merger enhances market power, then it is forbidden and it does not matter whether the acquired firm is failing or not. But economic theory shows that if the firm is failing, its acquisition does not harm competition and benefits market’s productive efficiency, maintaining productive assets which otherwise would be lost. Then, the interpreter faces a conflict between the action which is necessary to satisfy the statute (to omit the acquisition) and the action which seems to satisfy antitrust purposes (to acquire the failing firm).

A fresh solution intending to allow such acquisition would introduce an inconsistency in the antitrust system.

This problem, which shows a conflict between antitrust valid rules and its policy, was indeed solved by the courts. The innovative decision was enacted in *International Shoe Co. versus FTC (280 U.S. 291,302-303 (1930))*, when the Supreme Court held that acquisition of a failing company does not violate Section 7 of the *Clayton Act* (4, p.338), what means that "if the acquired firm is failing, an acquisition which tends to crate a monopoly is permitted". This interpretation motivated the *Celler-Kefauver Amendments* in 1950 making explicit the qualification in the text of the legal statute. Latter, following the Supreme Court interpretation in *Citizen Publishing Co. versus United States (394 U.S., 131, 138-139(1969))*, new relevant properties (conditions) were added by courts like "the ability of the failing firm to reorganize successfully" and "the presence of a viable alternative purchaser with less anticompetitive risk", which are now exceptions to the failing defense. These later changes were recognized by the *1992 Merger Guidelines* to the enforcement of antitrust law (4, p.338).

Antitrust jurists are supposed to interpret such authoritative decision and present theories about the proper solutions provided by the reconstructed normative system with reference to mergers in all relevant cases (including the hypothesis of failing firms). Such a comprehensive analysis will form the theoretical basis for new court decisions and amendments of the original statute. In order to model such reconstructions, we should then evaluate the systems resulting from external, internal and global refinement.

As a first approximation to the application of refinement operators to law, the standard deontic system SDL (cf.(6)) will be employed, leaving aside problems for the representation of conditionals or action in deontic logic. Let p stand for "market power", m for "merge" and f for "failing". The modal operator O represents obligation (Op means that p is obligatory, $O\neg p$ that p is forbidden and $\neg O\neg p$ that p is permitted). The capital letter S will represent the set of norms (formulas) contained in the literal interpretation of the Sherman and Clayton acts. It is assumed that theories about the statute are logically closed $S = Cn(S)$. The norm which is going to be challenged by refinement is that according to which "whenever the merger creates market power then it is forbidden", i.e. $p \rightarrow O\neg m \in S$.

If external refinement is applied, where f is the refuting condition, we obtain both that $(p \rightarrow O\neg m) \notin S \Cap \neg(p \rightarrow O\neg m)$ and $(f \rightarrow \neg(p \rightarrow O\neg m)) \in S \Cap \neg(p \rightarrow O\neg m)$. From e-closure it follows that $(f \rightarrow (p \wedge \neg O\neg m)) \in S \Cap \neg(p \rightarrow O\neg m)$ and hence $((f \wedge p) \rightarrow \neg O\neg m) \in S \Cap \neg(p \rightarrow O\neg m)$ and also $((f \wedge \neg p) \rightarrow \neg O\neg m) \in S \Cap \neg(p \rightarrow O\neg m)$. There is no indication about the normative solution if $\neg f \wedge p$ is the case. As the proposition $\neg p \rightarrow \neg O\neg m$ is not conflicting with the input and thus was not challenged by the new solution, it follows $(\neg p \rightarrow \neg O\neg m) \in S \Cap \neg(p \rightarrow O\neg m)$ [1]. Hence we have both $((f \wedge \neg p) \rightarrow \neg O\neg m) \in S \Cap \neg(p \rightarrow O\neg m)$ and $((\neg f \wedge \neg p) \rightarrow \neg O\neg m) \in S \Cap \neg(p \rightarrow O\neg m)$. The table below summarizes the reconstructed regulation.

[1] This follows from the upper bound property of AGM revision on which external refinement is based. According to this property, if a formula α does not by itself contradict the input (β), then there is at least one maximal subset Γ of the revised theory such that $\alpha \in \Gamma$ and $\Gamma \nvdash \beta$. So its easy to maintain α in the refined set assigning preference to those sets in the remainder which contain α.

Reconstruction A: external refinement.

Relevant Cases	Normative Solution
1. failing and market power	Permitted to merge
2. not failing and market power	(gap)
3. failing and not market power	Permitted merge
4. not failing and not market power	Permitted merge

Now, employing internal refinement, a sort of refined derogation is obtained, where instead of simply excluding the interpreted norm $p \rightarrow O\neg m$, only one of its consequences, namely the axiologically inadequate $f \rightarrow (p \rightarrow O\neg m)$ is extracted. By w-success we have $f \rightarrow (p \rightarrow O\neg m) \notin S \Supset (p \rightarrow O\neg m)$ and by w-preservation it follows $\neg f \rightarrow (p \rightarrow O\neg m) \in S \Supset (p \rightarrow O\neg m)$. The following reconstruction of the normative system would hold by application of internal refinement:

Reconstruction B: internal refinement

Relevant Cases	Normative Solution
1. failing and market power	(gap)
2. not failing and market power	Forbidden to merge
3. failing and not market power	Permitted merge
4. not failing and not market power	Permitted merge

In his later approach to deontic logic (first fully developed in (14)), von Wright argues that satisfiability (logical possibility to satisfy the norms) is the main standard of rationality to judge norm-giving-activity, what also applies to interpretive reconstruction of statutes. As the reconstructions of antitrust law by external and internal refinement provide consistent normative systems and an amendment based on either of them would create statutes free of contradictions, the resulting refined systems are rational.

However, externally or internally refined antitrust systems contain normative gaps and law professors, judges or lawyers would not hesitate to call the resulting regulation defective. For this reason it was argued in (12) that von Wright's standards of rationality should be improved by a standard of completeness, which would label irrational any law-giving act or interpretive reconstruction resulting in a system with at least one gap. According to the proposed standard, the lawgiver or the interpreter must define the deontic status of the action in all 2^n possible cases which are generated by the n conditions considered relevant. Now, using both satisfiability and completeness as rational standards, the proposed reconstructions by external and internal refinement would be irrational.

Let us then examine the application of the global refinement operator. By g-preservation and g-success we have both $\neg f \rightarrow (p \rightarrow O\neg m) \in S \circledcirc (p \rightarrow O\neg m)$ and $f \rightarrow \neg(p \rightarrow O\neg m) \in S \circledcirc \neg(p \rightarrow O\neg m)$. Hence, by g-closure, it follows $((\neg f \wedge p) \rightarrow O\neg m) \in S \circledcirc (p \rightarrow O\neg m)$ and $((f \wedge p) \rightarrow \neg O\neg m) \in S \circledcirc \neg(p \rightarrow O\neg m)$. The table below illustrates the result:

Reconstruction C: global refinement.

Relevant Cases	Normative Solution
1. failing and market power	Permitted to merge
2. not failing and market power	Forbidden to merge
3. failing and not market power	Permitted to merge
4. not failing and not market power	Permitted to merge

Thus, as global refinement preserve completeness, it has advantages over internal and external refinement as a model for the reconstruction of normative systems performed by legal dogmatic in order to justify exceptional normative solutions.

Ulrich Klug argued in (10, p.128) that it is reasonable to admit the contested validity of reasoning *a contrario sensu* when the condition for the normative solution has the strong sense of an exception. Accordingly, if an unpredicted property is to be considered relevant, then its presence should yield a different result than that in which it is absent. Note that in Reconstruction A, filling the gap in case 2 with the same normative solution as in case 1 would yield the irrelevance of the failing firm condition, and even the irrelevance of the presence or absence of market power. Mergers would be permitted whatever the circumstances might be. In Reconstruction B, by its turn, repeating the normative solution of case 2 in case 1 would amount to a redundant modification, in the sense that the resulting theory would be the same as that of literal statutory interpretation. The reconstruction provided by global refinement legitimates reasoning *a contrario*, since if the merger enhances market power, then the condition that the acquired firm is failing is both sufficient and necessary for its approval. It may be argued that such conditions are rarely adequate, since new unpredicted conditions may come along. But such a reconstruction may be acceptable as a provisory regulation, being sufficient to cope with legal cases until challenged by another refuting condition. Then the globally refined theory may be subject to further refinement of this sort.

Acknowledgements

I must thank Jorg Hansen for his criticism and suggestions on earlier versions, which motivated several modifications. I am grateful for the grant I receive from Fundação de Apoio à Pesquisa do Estado de São Paulo-FAPESP (99/02292-0).

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Appendix with Proofs:

Lemma 5. Suppose that $\alpha \in K$. By ct-success we have $\neg\beta \to \alpha \notin K \div \neg\beta \to \alpha$. Thus, by ct-closure, $\alpha \notin K \div \neg\beta \to \alpha$. By ct-recovery, $K \div \neg\beta \to \alpha \cup \{\neg\beta \to \alpha\} \vdash \alpha$ and thus $K \div \neg\beta \to \alpha \cup \{\beta\} \vdash \alpha$, since $\beta \vdash \neg\beta \to \alpha$. By deduction, we have that $K \div \neg\beta \to \alpha \vdash \beta \to \alpha$ and hence by ct-closure, $\beta \to \alpha \in K \div \neg\beta \to \alpha$.

Theorem 6. *i-closure* and *i-inclusion* follows immediately from ct-closure and ct-inclusion; for *i-recovery*, consider that, by ct-recovery, it holds that $K \subseteq (K \div c(\alpha)) + c(\alpha)$. Thus $K \subseteq (K \Supset \alpha) + c(\alpha)$.

Theorem 7. For the proof of *i-success*, suppose that $\nvdash \alpha$. If c is non-tautological then $\nvdash c(\alpha)$. Hence by ct-success $c(\alpha) \notin K \div c(\alpha) = K \Supset \alpha$; to prove *i-vacuity*, suppose that $K \nvdash \alpha$. If c is i-maximal, then $\vdash c(\alpha) \leftrightarrow \alpha$. It follows then from ct-extensionality and ct-vacuity that $K \div \alpha = K \div c(\alpha) = K \Supset \alpha = K$; for *i-extensionality*, suppose that $\vdash \alpha \leftrightarrow \gamma$. If c is extensional, we have $\vdash c(\alpha) \leftrightarrow c(\gamma)$ and by ct-extensionality, we obtain $K \Subset \alpha = K \div c(\alpha) = K \div c(\gamma) = K \Subset \gamma$.

Theorem 8. It is obvious that $\Supset$ satisfies ct-closure, ct-inclusion, ct-vacuity and also ct-extensionality as it satisfies the corresponding i-properties. For ct-success suppose that $\nvdash \alpha$. If c is non-tautological, then $\nvdash c(\alpha)$. By *i- success* it follows that $c(\alpha) \notin K \Supset \alpha$. Since c satisfies implication and $\Supset$ satisfies *i-closure* we have $\alpha \notin K \Supset \alpha$. Now we turn to ct-recovery. From *i-recovery* it follows that $K \subseteq (K \Supset \alpha) + c(\alpha)$, and as c satisfies implication, we have $K \subseteq (K \Supset \alpha) + \alpha$. Thus every internal refinement is a partial meet contraction. To prove that the reciprocal is false, consider the following partial meet contraction and observe that i-success does not hold: $Cn\{p, \neg q \to p\} \div p$.

Theorem 9. *g-closure* and *g-success* are immediate by construction. To prove *g-inclusion* we have that by i-inclusion $K \Supset \neg\alpha \subseteq K$ and, as c satisfies implication, we obtain $K \circledcirc \alpha = (K \Supset \neg\alpha) + c(\alpha) \subseteq K + \alpha$. For *g-consistency*, suppose that $\nvdash \neg\alpha$. Then it follows from i-success that $\neg\alpha \notin K \Supset \neg\alpha$ and by i-closure that $K \Supset \neg\alpha \nvdash \neg\alpha$. Hence, by implication of c, $K \Supset \neg\alpha \nvdash \neg c(\alpha)$. By classical logic we have that $K \Supset \neg\alpha \cup \{c(\alpha)\} \nvdash \neg c(\alpha)$ and thus $K \circledcirc \alpha \nvdash \neg(\beta \to \alpha)$, which means that $K \circledcirc \alpha \neq \mathcal{L}$. To prove *g-vacuity*, suppose that $K \nvdash \neg\alpha$, then by i-vacuity $K \Supset \neg\alpha = K$. If c is e-maximal, then it holds that $\vdash c(\alpha) \leftrightarrow \alpha$ and hence $K \circledcirc \alpha = (K \Supset \neg\alpha) + c(\alpha) = K + c(\alpha) = K + \alpha$. For the proof of *g-extensionality* $\vdash \alpha \leftrightarrow \gamma$ implies $\vdash c(\neg\alpha) \leftrightarrow c(\neg\gamma)$. Thus, $K \Supset \neg\alpha = K \Supset \neg\gamma$ and hence $K \circledcirc \alpha = K \circledcirc \gamma$. For *g-preservation*, note that if c is negation invariant, then for $c(\alpha) = \beta \to \alpha$ we have $c(\neg\alpha) = \beta \to \neg\alpha$ and thus by i-preservation we have that $(\neg\beta \to \neg\alpha) \in K \Supset \neg\alpha$. Hence $(\neg\beta \to \neg\alpha) \in (K \Supset \neg\alpha + c(\alpha)) = K \circledcirc \alpha$.