

# On the Structure and Soundness of AI & Law Objects

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## 1 Introduction

We present a (meta)theory that serves as a framework where case-based or statute-based models and systems can be understood, compared and prototyped. Central to our approach is the organization of theories under a notion of covering. This view helps to identify a disposition of elements based on a well-recognized underlying algebraic structure where theory definitions are reified and included in the same set together with their extensions and practical implementations.

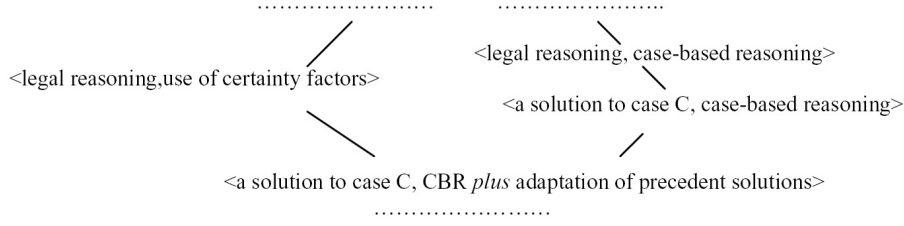
## 2 The Nature of AI & Law Objects and a Logic for their Construction

We believe AI & Law objects of study are not legal intelligent systems or models for legal systems *alone*: they are the *relation* established between a legal domain or system of concepts and a corresponding computational specification (a “translation” where a given system of concepts is somehow transformed in another one), relation we call reduction.

The set of possible reductions can be defined as the cartesian product  $Law \times AI_{\perp}$ .  $Law$  is a compound domain (i.e. it can be a cartesian product itself) encompassing a wide variety of denotable values referring to ideal, factual, prescriptive and descriptive phenomena: legislation, facts, cases, court decisions, *et cetera*.  $AI_{\perp}$  is a domain holding formal specifications [1] e.g. well-formed constructions established mainly at symbolic levels ( $AI_{\perp}$  means  $AI$  holds an “undefined” element). Stated as denotable values, elements in  $Law$  can be organized under a notion of *covering*, a relation organizing knowledge with respect to their “definedness”, i.e. with respect to their “knowledge content” [2] (for example:  $\langle name, age, course \rangle$  has less content than  $\langle “Bob”, 24, Logics \rangle$ ).  $AI_{\perp}$  specifications can also be ordered under their “definedness” [1] (e.g. a many-sorted signature with respect to a single-sorted signature holding just one of those sorts.) Provided this, we define

**Def. 1.** *Covering relation for reductions: Let  $r = \langle l, a \rangle \in Law \times AI_{\perp}$ . Let  $r', r'' \in Law \times AI_{\perp}$ . Let  $(Law, \leq), (AI_{\perp}, \leq)$  be covering relations based on the notion of “definedness” of denotational semantics over both  $Law$  and  $AI_{\perp}$ . Then  $(Law \times AI_{\perp}, \leq)$  is a covering relation over reductions if and only if  $\langle l, a \rangle \leq \langle l', a' \rangle$  implies  $l \leq l'$  or  $a \leq a'$  (or both) for every  $\langle l, a \rangle, \langle l', a' \rangle \in Law \times AI_{\perp}$ .*

Definition 1 says “more abstract” AI & Law objects (e.g. theories) are placed above their extensions, instantiations and practical implementations. Therefore under this denotational semantics view the same object is an instance of the object(s) above and a class of all objects located below (figure 1.) The  $\leq$  relation is an induced *partial order* over reductions as it verifies [3]: reflexivity ( $\langle l, a \rangle \leq \langle l, a \rangle$ ), antisymmetry ( $\langle l, a \rangle \leq \langle l', a' \rangle$  and  $\langle l', a' \rangle \leq \langle l, a \rangle$  imply  $\langle l, a \rangle = \langle l', a' \rangle$ ) and transitivity ( $\langle l, a \rangle \leq \langle l', a' \rangle$  and  $\langle l', a' \rangle \leq \langle l'', a'' \rangle$  imply  $\langle l, a \rangle \leq \langle l'', a'' \rangle$ ). Now let  $\wedge$  denote a generalization operation over denotational definitions (i.e. some knowledge is *removed*) and let  $\vee$  be a specialization operation over definitions (i.e. knowledge is *joint together* and/or *added*):



**Figure 1:** A sketch of a *Hasse diagram* for the reductions' poset.

then the following properties are easily verifiable on  $(Law \times AI_{\perp}, \leq)$ : commutativity ( $\langle l, a \rangle \vee \langle l', a' \rangle = \langle l', a' \rangle \vee \langle l, a \rangle$  and  $\langle l, a \rangle \wedge \langle l', a' \rangle = \langle l', a' \rangle \wedge \langle l, a \rangle$ ), associativity ( $\langle l, a \rangle \vee (\langle l', a' \rangle \vee \langle l'', a'' \rangle) = (\langle l, a \rangle \vee \langle l', a' \rangle) \vee \langle l'', a'' \rangle$  and  $\langle l, a \rangle \wedge (\langle l', a' \rangle \wedge \langle l'', a'' \rangle) = (\langle l, a \rangle \wedge \langle l', a' \rangle) \wedge \langle l'', a'' \rangle$ ), idempotence ( $\langle l, a \rangle \vee (\langle l', a' \rangle \vee \langle l', a' \rangle) = (\langle l, a \rangle \vee \langle l', a' \rangle) \vee \langle l', a' \rangle$  and  $\langle l, a \rangle \wedge (\langle l', a' \rangle \wedge \langle l', a' \rangle) = (\langle l, a \rangle \wedge \langle l', a' \rangle) \wedge \langle l', a' \rangle$ ) and absorption ( $\langle l, a \rangle = \langle l, a \rangle \vee (\langle l, a \rangle \wedge \langle l', a' \rangle)$  and  $\langle l, a \rangle = \langle l, a \rangle \wedge (\langle l, a \rangle \vee \langle l', a' \rangle)$ ). This way we identify an algebra for AI & Law objects in its own right. It can be seen as a logic of construction of AI & Law objects. The theory proposal method in [4] is an example of  $\wedge$ -use.

Comparisons among objects can be performed by analysing their relative positions in the poset. *Is-a* relationships can be discovered following paths in the poset.

### 3 The Interdisciplinary Validity Problem (IVP)

We are interested in AI outcomes to be accepted as *legal* answers, that is: “*outcomes must not only be checked for (specification) correctness but also confronted to expected legal results*” Stated this way, IVP establishes a relation connecting a “low level” AI interpreter and a “high level” legal interpreter. Given a reduction  $\langle l, a \rangle$ , given  $vm(Law), vm(AI_{\perp})$  available validation methods for *Law* and  $AI_{\perp}$  elements respectively, and being *eval* an evaluation relation, we have the following IVP schema

**Def. 2.**  $IVP(\langle l, a \rangle, vm(l), vm(a)) : eval(vm(l), l) \approx eval(vm(a), a); eval(vm(l), a)$

In IVP the relation among  $l$  and  $a$  through  $vm(l)$  is what counts. Def. 2. holds when the evaluation of  $l$  through legal methods agrees (under some criteria abstractly denoted  $\approx$ ) with outcomes obtained when evaluating  $a$  using the same methods used for  $l$ , given  $a$  correct.

### 4 Definition of a Reduction (meta)Theory

**Def. 3.** A *Reduction Theory* is a tuple:  $\langle (Law \times AI_{\perp}, \leq), vm(Law), vm(AI_{\perp}), IVP \rangle$

The Reduction Theory models AI & Law objects as relations among legal phenomena and specifications; models generalization/specialization relationships among reductions through an induced notion of covering, and it includes an IVP axiom schema (which we believe is a truly AI & Law-sensitive feature) that works as a tool for legal accuracy measurement.

## References

- [1] D. Schmidt. Denotational Semantics. A Methodology for Language Development. Allyn and Bacon, 1986, MA.
- [2] M. Kifer, G. Lausen. F-Logic: A Higher-Order Language for Reasoning about Objects, Inheritance and Scheme. ACM 1989.
- [3] S. Burris, H. Sankappanavar. A Course in Universal Algebra. Graduate Texts in Mathematics, Springer 1981.
- [4] T. Bench-Capon, G. Sartor. A Model of Legal Reasoning with Cases Incorporating Theories and Values. Artificial Intelligence. Kluwer, 2003.